Class: 11   Full Marks: 100
Subject: Mathematics   Time: 3:00 hours

Attempt all the questions.

Group: 'A' $(5 \times 3 \times 2 = 30)$

1. a. Construct truth table for $p \Rightarrow (p \lor q)$.

    b. Let $A = \{-1, 0, 2, 4, 6\}$ and a function $f : A \to R$ defined by $f(x) = \dfrac{x}{x+2}$. Find the range of f.

    c. Define even function with suitable example.

2. a. Prove that: $\tan^{-1} x + \tan^{-1} y = \tan^{-1}\left(\dfrac{x+y}{1-xy}\right)$

    b. Using principle of mathematical induction, prove that $1^3 + 2^3 + 3^3 + \ldots + n^3 = \left\{\dfrac{n(n+1)}{2}\right\}^2$.

    c. If $A = \begin{bmatrix} 3 & -5 \\ 4 & 6 \end{bmatrix}$, and $B = \begin{bmatrix} 0 & 3 \\ 1 & -2 \end{bmatrix}$, then find $(AB)^T$.

3. a. Solve by row equivalent method.
        $3x + 2y = -9$ and $2x - 3y = -6$

    b. If $x - iy = \sqrt{\dfrac{1-i}{1+i}}$, prove that $x^2 + y^2 = 1$.

    c. If one root of the equation $x^2 - px + q = 0$ be twice of the other root, show that $2p^2 = 9q$.

4. a. Find the equation of line through (5, 4) and perpendicular to the line $4x - 3y = 10$.

    b. Find the equation of circle whose centre is at (-4, 5) and a tangent is $3x - 4y + 2 = 0$.

    c. Evaluate: $\lim_{x \to a} \dfrac{\sqrt{2x} - \sqrt{3x-a}}{\sqrt{x} - \sqrt{a}}$

5. a. Find $\dfrac{dy}{dx}$ for $x^2 y^2 = x^2 + y^2$.

    b. Determine where the graph is concave upwards and where it is concave downward for $f(x) = x^4 - 8x^3 + 18x^2 - 24$.

    c. Evaluate: $\displaystyle\int \dfrac{dx}{1 - \sin x}$

Group: 'B' $(5 \times 2 \times 4 = 40)$

6. a. Define difference of sets. For non-empty subsets A and B of universal set U, prove that;
        $A - (B \cup C) = (A - B) \cup (A - C)$.
                OR
        Define absolute value of real number. For any two real numbers x and y, prove that
        $|x + y| \le |x| + |y|$.

    b. Sketch the graph of $y = x^2 + 2x + 3$ indicating its characteristics.

7. a. Solve: $\sin^2 \theta - 2\cos\theta + \dfrac{1}{4} = 0$
                OR

In any triangle ABC, prove that: $\tan\frac{1}{2}(B-C)=\frac{b-c}{b+c}\cot\frac{A}{2}$

b. Prove that: $\begin{vmatrix} 1+x & 1 & 1 \\ 1 & 1+y & 1 \\ 1 & 1 & 1+z \end{vmatrix}=xyz\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}+1\right)$

8. a. Solve the following equations by using Cramer's rule.

$x+2y-z=-5$

$2x-y+z=6$

$x-y-3z=-3$

b. Prove that a quadratic equation cannot have more than two roots.

9. a. Find the equations of the tangents drawn from the point (11, 3) to the circle $x^2+y^2=65$.

b. Evaluate: $\lim_{x \to \theta}\frac{x\sin\theta-\theta\sin x}{x-\theta}$

OR

A function f(x) is defined as:

$f(x)=\begin{cases} 2x-3 \text{ for } x<2 \\ 2 \text{ for } x=2 \\ 3x-5 \text{ for } x>2 \end{cases}$

Is the function f(x) continues at x=2? If not, how can the function f(x) be made continuous at x=2?

10. a. Find from first principle, the derivative of $\sqrt{\sin 2x}$.

b. Find area of ellipse $\frac{x^2}{9}+\frac{y^2}{16}=1$ using integration.

OR

Evaluate: $\int e^{ax}\cos bx$

Group: 'C' $(5\times6=30)$

11. Define domain and range of function. Also find the domain and range of $f(x)=x^2-6x+6$.

12. The sum of three numbers in A.P. is 36. When the numbers are increased by 1, 4, 43 respectively, the resulting numbers are in G.P. Find the numbers.

13. State De-Moivre's theorem. Use it to find square root of $4+4\sqrt{3}i$.

14. Find the equation to the pair of lines joining the origin to the intersection of the straight line $y=mx+c$ and the curve $x^2+y^2=a^2$. Prove that they are at right angles if $2c^2=a^2(1+m^2)$.

OR

Find the length of perpendicular drawn from the point (x₁, y₁) on line $Ax+By+C=0$.

15. A window is in the form of a rectangle surmounted by a semi-circle. If the total perimeter is 9 meters, find the radius of semi-circle for the greatest window area.

OR

Water flows into an inverted conical tank at the rate of 42 cm³/sec. When the depth of the water is 8 cm, how fast is the level rising? Assume that the height of the tank is 12 cm and the radius of the top is 6 cm.

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