Class: 12   Full Marks: 100
Subject: Mathematics   Time: 3:00 hours

Answer all the questions.

GROUP A

1.   a.   From 10 persons, in how many ways can a selection of 4 be made when two particular persons are always included?   [2]

     b.   Prove that $\dfrac{2}{1!}+\dfrac{4}{3!}+\dfrac{6}{5!}+\ldots=e.$

          [2]

     c.   Let G={0, 1, 2}. Form a composition table for G under addition module 3. Find the inverse element of 2.

2.   a.   Find the equation of the hyperbola with vertex (8, 0) and passing through the point $\left(8\sqrt{2},4\right).$

          [2]

     b.   Find the equation of the plane which makes equal intercepts on the axes and passes through the point (2, 3, 4).   [2]

     c.   ABCD is a parallelogram. G is the point of intersection of its diagonals and if O is any point. Show that $\overrightarrow{OA}+\overrightarrow{OB}+\overrightarrow{OC}+\overrightarrow{OD}=40\overrightarrow{G}.$ [2]

3.   a.   If $|\vec{a}+\vec{b}\vee \dot{\iota} \vee \vec{a}-\vec{b}\vee \dot{\iota},$ prove that $\vec{a}$ is perpendicular to $\vec{b}.$ [2]

     b.   Find the equation of the tangent to the curve $y=2x^{3}-5x^{2}+8$ at (2, 4).   [2]

     c.   Evaluate: $\displaystyle\int \frac{dx}{\sqrt{(x-\alpha)(x-\beta)}},(\beta>\alpha)$ [2]

4.   a.   Solve the differential equation $\dfrac{dy}{dx}=e^{x+y}+3x^{2}e^{y}.$ [2]

     b.   IF $n=15,\sigma x=3.2,\sigma y=3.4$ and $\sum(X-\acute{X})(Y-\acute{Y})=122$, find the correlation coefficient between the two variables.   [2]

     c.   A dice is thrown 3 times. Getting a 2 or 3 is numbered as a success. Find the probability of getting two successes.   [2]

5.   a.   If 6 persons where invited for a party, in how many ways can they and host be seated at a circular table? In how many ways they can be seated if two particular persons must be seated on either side of the host?   [4]

               OR

          A committee of 5 is to be selected from among 6 boys and 5 girls. Determine the number of ways of selecting the committee if it is to consist of at least 1 boy and 1 girl.

     b.   Show that {2^{n}:n∈Z} is an Abelian group with respect to multiplication.   [4]

6.   a.   Prove that the line $3x+4y+6=0$ is tangent to the parabola $2y^{2}=9x.$ Find its point of contact.   [4]

               OR

          Find the equation of the ellipse whose distance between two foci is 8 and the semi-latus rectum is 6.

     b.   Find the equation of the plane passing through the points $(1,1,0),(-2,2,-1)$ and $(1,2,1).$ [4]

7.   a.   Evaluate: $\displaystyle\int \frac{dx}{1-2\cos x}$

          [4]

     b.   Solve: $\left(1+x^{2}\right)\dfrac{dy}{dx}+2xy=4x^{2}$ [4]

OR

$$sinx\frac{dy}{dx}+(cosx).y=sinx.cosx$$

8. a. Consider the following distribution:

	Distribution A	Distribution B
Arithmetic Mean	100	90
Median	90	80
Standard Deviation	10	10

Is the Distribution A same as the Distribution B regarding the degree of variation and skewness? [4]

b. The probability of hitting a target is found to be 0.25. If eight hits are made, find the probability that (i) none will hit the target (ii) exactly two will hit the target. [4]

9. If $(1+x)^n=c_0+c_1 x+c_2 x^2+\ldots+c_n x^n$, prove that $c_0^2+c_1^2+c_2^2+\ldots+c_n^2=\frac{2n!}{(n!)^2}$. [6]

10. Define vector product of two vectors. Prove by vector method that in any triangle $ABC, \frac{a}{sinA}=\frac{b}{sinB}=\frac{c}{sinC}$.

11. State mean value theorem. Interpret it geometrically. Verify the mean value theorem for the function $f(x)=x(x-1)^2$ in [0, 2]. [6]

OR

Find from first principle, the derivative of $sin^{-1}x$.

GROUP C

12. a. Draw the graph of the following inequalities. [2]
   $3x+4y\le24, 0\le y\le4, 0\le x\le7$ and shade the feasible region.

   b. Convert the hexadecimal number $22F_{16}$ into binary form. [2]

   c. Examine whether the system of equations $x+3y-2z=0, 2x-3y+z=1$, and $4x-3y+z=3$ is diagonally dominant or not? [2]

13. a. Solve using Gauss elimination method, the following equations. [4]
   $x+3y-2z=5, 3x+5y+6z=7, 2x+4y+3z=8$

   OR

   Solve the following system of equation using Gauss Seidal method:
   $3x_1+x_2=5, x_1+2x_2=5$

   b. Using Simpson's $\frac{1}{3}$ rule, evaluate $\int_0^1 \frac{dy}{1+x}$ for n=4 correct to 4 places of decimal. [4]

14. A furniture company produces chairs and tables. Each table takes four hours of carpentry department and two hours of finishing. Each chair requires three hours of carpentry and one hour of finishing. During the current week, 240 hours of carpentry time are available and 100 hours of finishing time. Each table produced gives a profit of Rs.70 and each chair a profit of Rs.50. How many chairs and tables should be made for maximum profit? [6]

15. Use Bisection method to find the solution accurate to within 10^{-2} for $x^3-7x^2+14x-6=0$ on [0, 1]. [6]

OR

Write three methods for measuring error. Approximate $\sqrt{11}$ by Newton-Raphson's method with accuracy 0.00001.

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