

# Mathematics

## Grade: XI

*Full Marks: 100*  
*Teaching hours: 150*

### **I. Introduction:**

This course deals with the fundamentals of advanced mathematical concepts. It also tries to consolidate the concepts and skills learnt in Mathematics course in school level. It is desirable at the end of each unit sufficient related problems be solved.

### **II. Specific Objectives:**

On completion of this course students will be able to:

1. use principles of elementary logic to find the validity of statement;
2. state field and order axioms of Real number system;
3. define functions and illustrate them graphically;
4. sketch the curves;
5. use trigonometrical relations to find the general values, understand inverse circular functions and their properties and to find property & solution of triangle;
6. state properties of A.S., G.S. and H.S. Understand infinite series and use method of mathematical induction to establish the result;
  
7. define transpose, adjoint and inverse of matrix, state properties of determinants;
8. use matrix and determinant to solve system of linear equations;
9. explain the idea of a complex number, verify their properties, prove De-Moivre's theorem and use it;
10. define polynomial equations, establish fundamental theorem of algebra and quadratic equation, and find relation between roots and coefficients of a quadratic polynomials;
11. define straight lines, pair of lines in terms of co-ordinates and establish their properties;
12. define circle in terms of coordinates and establish their properties;
13. define limit of a function, establish properties of limits;
14. define continuity of a function using the concept of limit;
15. define derivative of a function and give its geometrical interpretation as rate of change;
16. use derivative to determine the nature of the function and determine the maxima and minima of a function and apply differentiation to find tangent & normal, increasing & decreasing function;
17. define antiderivative as an inverse process of derivative and use various methods of integration; and
18. define integration as the area of the sum, and apply definite integral to find the area between the curves.

### III. Course Contents:

#### Unit 1: Sets, Real Number System and Logic 10 hrs

**Sets:**

Sets and set operations, Theorems based on set operations.

**Real Number System:**

Real numbers, Field axioms, Order axioms, Interval, Absolute value, Geometrical representation of the real numbers.

**Logic:**

Introduction, statements, Logical connectives, Truth tables, Basic laws of logic.

#### Unit 2: Relations, Functions and Graphs 12 hrs

**Relations:**

Ordered pair, Cartesian product, Geometrical representation of Cartesian product, relation, Domain and range of a relation, Inverse of a relation.

**Functions:**

Definition, Domain and range of a function, Functions defined as mappings, Inverse function, Composite function, functions of special type (Identity, Constant, Absolute value, Greatest integer), Algebraic (Linear, quadratic and cubic), Trigonometric, Exponential logarithmic functions and their graphs.

#### Unit 3: Curve Sketching 10 hrs

Odd and even functions, Periodicity of a function, symmetry (about x – axis, y – axis and origin) of elementary functions, Monotonicity of a function, Sketching graphs of

polynomial functions  $\left(\frac{1}{x}, \frac{x^2 - a^2}{x - a}, \frac{1}{x + a}, x^2, x^3\right)$ , Trigonometric, exponential,

logarithmic functions (Simple cases only)

#### Unit 4: Trigonometry 10 hrs

Inverse circular functions, Trigonometric equations and general values, properties of a triangle (sine law, Cosine law, tangent law, Projection laws, Half angle laws), the area of a triangle. Solution of a triangle (simple cases)

#### Unit 5: Sequence and Series, and Mathematical Induction 12 hrs

**Sequence and Series:**

Sequence and series, type of sequences and series (Arithmetic, Geometric, Harmonic), Properties of Arithmetic, Geometric, and Harmonic sequences, A.M., G.M. And H.M. Relation among A.M., G.M. and H.M., Sum of infinite geometric series.

**Mathematical Induction:**

Sum of finite natural numbers, Sum of the squares of first n – natural numbers, Sum of cubes of first n – natural numbers. Intuition and induction, principle of mathematical induction.

**Unit 6: Matrices and Determinants****8 hrs**

Matrices and operation on matrices (Review), Transpose of a matrix and its properties, Minors and Cofactors, Adjoint, Inverse matrix. Determinant of a square matrix, properties of determinants (Without proof) upto  $3 \times 3$ .

**Unit 7: System of Linear Equations****8 hrs**

Consistency of system of linear equations, solution of a system of linear equations by Cramer's rule, Matrix method (row – equivalent and Inverse) upto three variables.

**Unit 8: Complex Number****12 hrs**

Definition of a complex number, Imaginary unit, Algebra of complex numbers, Geometric representation of a complex number, Conjugate and absolute value (Modulus) of a complex numbers and their properties, Square root of a complex number, Polar form of a complex number, product and Quotient of complex numbers. De Moivre's theorem and its application in finding the roots of a complex number, properties of cube roots of unity.

**Unit 9: Polynomial Equations****8 hrs**

Polynomial function and polynomial equations, Fundamental theorem of algebra (without proof), Quadratic equation Nature and roots of a quadratic equation, Relation between roots and coefficients, Formation of a quadratic equation, Symmetric roots, one or both roots common.

**Unit 10: Co-ordinate Geometry****12 hrs****Straight line:**

Review of various forms of equation of straight lines, Angle between two straight lines, condition for parallelism and perpendicularity, length of perpendicular from a given point to a given line, Bisectors of the angles between two straight lines.

**Pair of lines:**

General equation of second degree in x and y, condition for representing a pair of lines, Homogeneous second degree equation in x and y, Angle between pair of lines, Bisectors of the angles between pair of lines.

**Unit 11: Circle****10 hrs**

Equation of a circle in various forms (Centre at origin, centre at any point, general equation of a circle, circle with a given diameter), Condition of Tangency of a line at a point to the circle, Tangent and normal to a circle.

**Unit 12: Limits and Continuity****10 hrs**

Limits of a function, Indeterminate forms, Algebraic properties of limits (without proof), Theorem on limits of algebraic, Trigonometric, Exponential and logarithmic functions

functions  $\left( \lim_{x \rightarrow a} \frac{x^n - a^n}{x - a}, \lim_{x \rightarrow 0} \frac{\sin x}{x}, \lim_{x \rightarrow 0} \frac{e^x - 1}{x}, \lim_{x \rightarrow 0} \frac{\log(1+x)}{x} \right)$

Continuity of a function, Types of discontinuity, Graph of discontinuous function.

**Unit 13: The Derivatives**

**8 hrs**

Derivative of a function, Derivatives of algebraic, trigonometric, exponential and logarithmic functions by definition (simple forms), Rules of differentiation, Derivatives of parametric and implicit functions, Higher order derivatives.

**Unit 14: Applications of Derivatives**

**12 hrs.**

Geometric interpretation of derivative, Monotonocity of a function, Interval of monotonocity, Extrema of a function, Concavity, Points of inflection, Derivative as rate measure.

**Unit 15: Antiderivatives and its Applications**

**10 hrs**

Antiderivative, Integration using basic integrals, Integration by substitution and by parts method, the definite integral, The definite integral as an area under the given curve, Area between two curves.

**IV. Evaluation Scheme:**

<u>No. of questions</u>	<u>Marks</u>	<u>Total</u>	<u>Remarks</u>
15	2	30	covering all units.
10	4	40	with four OR-questions from the same.
5	6	30	with two OR-questions from the same.

The questions of 6 marks will be asked from the units with 12 or more credit hours.

**V. Reference books:**

1. Adhikari, D.B. and et.al. *Element of Mathematics Part I*. Himaly Book stall.
2. Bajracharya, D.R. ; Shrestha, R.M. and et.al. *Higher Secondary Level Basic Mathematics ( For Grade XI)*. Kathmandu: Sukunda Pustak Bhawan.
3. Bajracharya, P.M. and Basnet, G. (2008). *Fundamentals of Mathematics for Grade XI*. Kathmandu: Buddha Academic Publishers & Distributors P. Ltd.
4. Koirala, S. and et.al. *Fundamentals of Mathematics*. Kathmandu: Nepal Sahitya Prakashan Kendra.
5. Pant, S.R. and et.al. *A Text-Book of Higher Secondary Mathematics (For Grade XI)*. Kathmandu: Buddha Academic Publishers and Distributors P. Ltd.
6. Uprety, K.N. and Ghimire, K.P., *Foundation of Mathematics, (For Grade XI)*. Pigeon Educational Publisher.

**Mathematics**  
**Grade - XI**  
**Group 'A' [5 × 3 × 2]**

Attempt ALL questions.

1. a) Write truth table for  $p \wedge q \Leftrightarrow p \wedge q$ , hence draw a conclusion from the truth table.  
b) If  $a = \{1, 2, 3\}$  find the relation on  $A$  satisfying the condition  $x + y < 4$ . Is this relation a function? Give reason.  
c) Test periodicity and symmetricity of the function  $y = \cos x$ .
2. a) Find the value of  $x$  for which  $\sin x = \frac{1}{2}$  and  $\cos x = \sqrt{\frac{3}{2}}$  ( $0 \leq x \leq 2\pi$ ).  
b) Using mathematical induction, prove that  $1 + 3 + 5 + \dots + n = n^2$ .  
c) Find the inverse  $\begin{pmatrix} 3 & 2 \\ -1 & 6 \end{pmatrix}$ .
3. a) Solve the following system of linear equations by inverse matrix method, if possible:  
$$5x - 3y = 9$$
$$10x - 6y = 16$$
  
b) Find the value of the real numbers  $x$  and  $y$  if  $(x + z) + yt = (3 + t)(1 + 2t)$ .  
c) Determine the nature of the roots of the equation  $2x^2 + 3x - 2 = 0$ .
4. a) Show that the points  $(1, 2)$  and  $(2, -3)$  lie on the opposite side of the line  $5x - 2y - 3 = 0$ .  
b) Find the equation of the circle concentric with  $x^2 + y^2 - 8x - 12y + 14 = 0$ , and passing through  $(5, 4)$ .  
c) Find the limit of  $f(x) = \frac{x^2 - 4}{x - 2}$  as  $x \rightarrow 2$ . Is  $f(x)$  continuous? If not, find the point of discontinuity.
5. a) Find the derivative of  $\sec^2(\tan \sqrt{x})$ .  
b) For any curve  $y = f(x)$ , what do  $f'(x) > 0$  and  $f'(x) < 0$  represent?  
c) Evaluate:  $\int \frac{x}{(1-x^2)^{3/2}} dx$ .

**Group 'B' [5 × 2 × 4]**

6. a) If  $A, B$  and  $C$  are any three non-empty sets, prove that  
$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

OR

Define the absolute value of a real number. If  $a$  is any positive real number and  $x \in \mathbb{R}$ , prove that  $|x| < a \Leftrightarrow -a < x < a$ .  
b) Sketch the graph of  $y = x^2 - 6x + 9$  indicating its different characteristics.
7. a) If  $\cos^{-1} x + \cos^{-1} y + \cos^{-1} z = \pi$ , Allow that  $x^2 + y^2 + z^2 + 2xyz = 1$ .

OR

State and prove sine law.

b) Prove that 
$$\begin{bmatrix} a & b & c \\ a & b^2 & c^2 \\ b+c & c+a & a+b \end{bmatrix} = (b-c)(c-a)(a-b)(a+b+c).$$

8 a) Applying Cramer's rule or row equivalent method, solve the system of linear equations :

$$3x + y + 2z = -1$$

$$2x + 3y + z = 5$$

$$x + 2y - z = 8$$

b) If the equation  $x^2 + px + q = 0$  and  $x^2 + qx + p = 0$  have a common root, prove that either  $p = q$  or  $p + q + 1 = 0$ .

9 a) Prove that the line  $y = mx + c$  is tangent to the circle  $x^2 + y^2 = a^2$  if  $c = \pm a\sqrt{1+m^2}$ . Also show that  $3x + 4y = 20$  is tangent to the circle  $x^2 + y^2 = 16$ .

b) Evaluate:  $\lim_{x \rightarrow \theta} \frac{x \cot \theta - \theta \cot x}{x - \theta}$ .

OR

A function  $f(x)$  is defined as follows:

$$f(x) = \begin{cases} 2x-3 & \text{for } x < 2 \\ 2 & \text{for } x = 2 \\ 3x-5 & \text{for } x > 2 \end{cases} .$$

Is  $f(x)$  continuous at  $x = 2$ . If not, how can  $f(x)$  be made continuous at  $x = 2$ .

10. a) Find the derivative of  $\sqrt{3-2x}$  from first principles.  
 b) Find area between the curves  $y^2 = 4ax$  and  $x^2 = 4ay$ .

Group 'C' [5 × 6 = 30]

11. Define domain and range of a function. Find the domain and the range of the  $f(z) = \sqrt{21-4x-x^2}$ .

12. If  $AM, GM$  and  $HM$  be the arithmetic, geometric and harmonic means between two unequal positive numbers prove that:

$$\text{i) } GM^2 = AM \times HM \quad \text{ii) } AM > GM > HM$$

13. Derive the formula for the length of the perpendicular from a point  $(x, y)$  to a line  $x \cos \alpha + y \sin \alpha = p$ . Also, find the distance between the parallel lines  $5x - 12y + 8 = 0$  and  $10x + 24y - 3 = 0$ .

Or

Find the condition that the general equation of second degree may represent a line-pair. If  $3x^2 + 5xy - 3y^2 + 2x + 3y = 0$  represents a line pair, show that the lines are perpendicular.

14. Define conjugate of a complex number. Find the square root of  $\frac{2-36i}{2+3i}$ .

15. Find the local maxima and minima, and also the point of inflection (if exists) of the function  $f(x) = 4x^3 - 6x^2 - 9x + 1$  on the interval  $(-1, 2)$ . Also, examine whether the function is increasing or decreasing at  $x = 0$ .

Or

Two concentric circles are expanding in such a way that the radius of the inner circle is increasing at the rate of  $8\text{ cm/sec}$  and that of the outer circle at the rate of  $5\text{ cm/sec}$ . At a certain instant the radii of the inner and outer circles are respectively  $24\text{ cm}$  and  $30\text{ cm}$ . At what rate does the area between the two circles change?

# Mathematics

## Grade: XII

*Full Marks: 100*

*Teaching hours: 150*

### **I. Introduction:**

This course is a continuation to the course of Grade XI. It further consolidates the concept learnt in Grade XI. For the completeness of the course it includes other areas in mathematics such as Mechanics, Group theory, Statistics and probability, Numerical methods.

Group (A) contains eleven units, and is compulsory while Group (B) and Group (C) each containing four units are optional. Students will be required to offer either Group (B) or Group (C).

### **II. Specific Objectives:**

On completion of this course students will be able to:

1. state basic principles of counting and find number of permutations and combinations of set of objects with various conditions;
2. prove binomial theorem for positive index, state exponential and logarithmic series, and apply them in solving problems;
3. understand group as algebraic structure and establish simple results on finite and infinite groups;
4. derive equations of parabola, ellipse and hyperbola, and find tangent & normal to the parabola;
5. locate points in space and derive the equation of plane;
6. define product of vectors and give their geometrical meaning and use it to find various results of geometry and trigonometry;
7. establish the relation between continuity and differentiability of a function, compute, derivatives of exponential, logarithmic and hyperbolic and inverse circular functions, apply Hospital's rule;
8. determine standard integrals, use partial fractions of integrate rational function;
9. define differential equations and different forms of solutions and use them in application;
10. state measures of dispersion and find coefficient of correlation and equation of regression;
11. define probability, establish basic laws of probability;
12. define parallelogram of forces, composition and resolution of forces, triangle of forces, and prove Lami's theorem;
13. find resultant of like and unlike parallel forces, moment of a force and moment of couple of forces;
14. state and use Newton's laws of motion. Find Impulse, work, Energy & Power, and acquaint with a projectile;
15. formulate linear programming problem, solve LPP graphically and by simplex method;



16. determine a root of equations by numerical methods; and
17. evaluate integrals by trapezoid and Simpson's rules.

### **III. Course Contents:**

#### **Group 'A'**

##### **Unit 1: Permutation and Combination.**

**10 hrs**

Basic principle of counting, Permutation of (a) set of objects all different (b) set of objects not all different (c) circular arrangement (d) repeated use of the same object. Combination of things all different, Properties of combination.

##### **Unit 2: Binomial Theorem**

**10 hrs**

Binomial theorem for a positive integral index, general term. Binomial coefficients, Binomial theorem for any index (Without proof), Application to approximation, Euler's number. Expansion of  $e^x$ ,  $a^x$  and  $\log(1+x)$  (without proof).

##### **Unit 3: Elementary Group Theory**

**8 hrs.**

Binary operation, Binary operation on sets of integers and their properties, Definition of a Group, Groups whose element are not numbers, Finite and infinite groups, Uniqueness of identity, Uniqueness of inverse, Cancellation law, Abelian Group.

##### **Unit 4: Conic Sections**

**12 hrs**

Standard equation of parabola, Ellipse and Hyperbola, Equations of tangent and normal to a parabola at a given point.

##### **Unit 5: Co-ordinates in Space**

**12 hrs**

Co-ordinate axes, Co-ordinate planes, The octants, Distance between two points, External and internal point of division, Direction cosines and ratios, fundamental relation between direction cosines, Projections, Angle between two lines. General equation of a plane, Equation of a plane in intercept and normal form, Plane through three given points, Plane through the intersection of two given planes, Parallel and perpendicular planes, angle between two planes distance of a point from a plane.

##### **Unit 6: Vectors and its Applications**

**14 hrs**

Cartesian representation of vectors, Collinear and non-collinear vectors, Coplanar and non-Coplanar vectors, Linear combination of vectors. Scalar product of two vectors, Angle between two vectors, Geometric interpretation of scalar product, Properties of Scalar Product, Condition of perpendicularity. Vector product of two vectors, Geometric interpretation of vector product, Properties of Vector Product, Application of product of vectors in plane trigonometry.

**Unit 7: Derivative and its Application** **14 hrs**  
Derivative of inverse trigonometric, exponential and logarithmic functions by definition, Relationship between continuity and differentiability, Rules for differentiating hyperbolic function and inverse hyperbolic function, Composite function and function of the type  $f(x)^{g(x)}$ .  
L'Hospital's rule (for  $0/0$ ,  $\infty/\infty$ ), Differentials, Tangent and Normal, Geometric interpretation and application of Rolle's theorem and Mean value theorem.

**Unit 8: Antiderivatives** **7 hrs**  
Antiderivatives, Standard integrals, Integrals reducible to standard forms, Integrals of rational functions.

**Unit 9: Differential Equations and their Applications** **7 hrs**  
Differential equation and its order and degree, Differential equations of first order and first degree: Differential equations with separable variables, homogeneous and exact differential equations.

**Unit 10: Dispersion, Correlation and Regression** **12 hrs**  
Dispersion, Measures of dispersion (Range, Semi interqrtil range, Mean deviation, Standard deviation) variance, Coefficient of variation, Skewness, Karl Pearson's and Bowley's Coefficient of Skewness, Bivariate distribution, Correlation, Nature of correlation, Correlation coefficient by Karl Pearson's method. Interpretation of correlation coefficient, Properties of correlation coefficient (Without proof)  
Regression equation, Regression line of  $y$  on  $x$  and  $x$  on  $y$ .

**Unit 11: Probability** **8 hrs**  
Random experiment, sample space, Event, Equally likely cases, Mutually exclusive events, Exhaustive cases, Favourable cases, Independent and dependent cases, Mathematical and empirical definition of probability, Two basic laws of probability, Conditional probability (without proof), Binomial distribution, Mean and Standard deviation of binomial distribution (without proof).

### **Group 'B'**

**Unit 12: Statics** **9 hrs.**  
Forces and resultant forces, Parallelogram of forces, Composition and resolution of forces, Resultant of coplanar forces acting at a point, Triangle of forces and Lami's theorem.

**Unit 13: Statics (Continued)** **9 hrs**  
Resultant of like and unlike parallel forces, Moment of a force, Varignon's theorem, Couple and its properties (without proof).

**Unit 14: Dynamics** **9hrs**  
Motion of particle in a straight line, Motion with uniform acceleration, Motion under gravity, Motion down a smooth inclined plane. The concepts and theorems be restated and formulated as application of calculus.

**Unit 15: Dynamics (Continued)** **9 hrs**  
Newton's laws of motion, Impulse, Work, Energy and Power, Projectiles.

**Group 'C'**

**Unit 16: Linear Programming** **11 hrs**  
Introduction of a linear programming problem (LPP), Graphical solution of LPP in two variables, Solution of LPP by simplex method (two variables).

**Unit 17: Computational Method** **9 hrs**  
Introduction to Numerical computing (Characteristics of Numerical computing Accuracy, Rate of Convergence, Numerical Stability, Efficiency); Number systems (Decimal, Binary, Octal & Hexadecimal system conversion of one system into another), Approximations and error in computing Roots of nonlinear equation, Algebraic, polynomial & transcendental equations and their solution by bisection and Newton – Raphson Methods,

**Unit 18: Computational Method (Continued)** **8 hrs**  
Solution of system of linear equations by Gauss elimination method, Gauss-Seidel method, Ill conditioned systems, Matrix inversion method.

**Unit 19: Numerical Integration** **8 hrs**  
Trapezoidal and Simpson's rules, estimation of errors.

**IV. Evaluation Scheme:**

<u>No. of questions</u>	<u>Marks</u>	<u>Total</u>	<u>Remarks</u>
15	2	30	covering all units.
10	4	40	with four OR-questions from the same.
5	6	30	with two OR-questions from the same.

The questions of 6 marks will be asked from the units with 12 or more credit hours.

## V. Reference books:

1. Adhikari, D.B. and et.al. *Element of Mathematics Part II*. Himaly Book stall.
2. Bajracharya, D.R.; Shrestha, R.M. and et.al. *Higher Secondary Level Basic Mathematics ( For Grade XII)*. Kathmandu: Sukunda Pustak Bhawan.
3. Bajracharya, P.M. and Basnet, G. (2008). *Fundamentals of Mathematics (For Grade XII)*. Kathmandu: Buddha Academic Publishers and Distributors P. Ltd.
4. Balagurusamy, E., *Numerical Methods*. India: Tata Mc Graw Hill.
5. Pant, S.R. and et.al. *A Text-Book of Higher Secondary Mathematics (For Grade XII)*. Kathmandu: Buddha Academic Publishers and Distributors P. Ltd.
6. Ranganath, G.K. and Narayan, B.S. *A Text-Book of Computer Oriented Numerical Methods and Linear Programming*.
7. Upadhyaya, M.P., *An Introduction to Linear Programming*. kathmandu: Sukunda Pustak Bhawan.
8. Uprety, K.N. and Ghimire, K.P., *Foundation of Mathematics, (For Grade XII)*. Pigeon Educational Publisher.
9. Sitaula, K., Sharma, B., Bhatta, C.R., *Essential Mathematics*